Bee Colony Optimization (BCO)
The first fifteen years

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Presentation outline

BCO

- Optimization framework, meta-heuristic method;
- Nature-Inspired Algorithm;
- Population based method;
- Imitates swarm behavior;
- Explores collective (swarm) intelligence;
- Based on foraging behavior of honeybees;
- Proposed by Lučić and Teodorović, 2001.

Other bees foraging algorithms

- Artificial Bee Colony (ABC)
  [1] Karaboga, D., "An idea based on honey bee swarm for numerical optimization", Technical report, Erciyes University, Engineering Faculty Computer Engineering Department Kayseri/Turkiye, (2005).
  [2] Karaboga, D., and Basturk, B., "A powerful and efficient algorithm for numerical function optimization: artificial bee colony (ABC) algorithm", Journal of global optimization, 39(3), (2007), 459-471.
- Bees Algorithm (BA)
  [1] Pham, D. T., Ghanbarzadeh, A., Koc, E., Otri, S., and Zaidi, M., "The bees algorithm - a novel tool for complex optimisation problems", Proc. 2nd Virtual International Conference on Intelligent Production Machines and Systems (IPROMS 2006), Elsevier, Cardiff, Wales, UK, (2006) 454-459.
  [2] Pham, D., T., Soroka, A. J., Ghanbarzadeh, A., and Koc, E., "Optimising neural networks for identification od wood defects using the bees algorithm", Proc. IEEE International Conference on Industrial Informatics, Singapore, (2006) 1346-1351.

Bees in the nature

[1] S. Camazine, and J. Sneyd, "A model of collective nectar source by honey bees: Self-organization through simple rules", J. Theor. Biol. vol. 149, 1991, pp. 547-571.

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- They return to the hive and opt to one of the possibilities:
  1. become *recruiters*, i.e. to dance and inform their hive-mates about locations (directions and distances), quantities, and qualities of the available food sources;
  2. return to the discovered nectar source and continue collecting nectar;
  3. abandon the food location and become *uncommitted followers*.

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- Followers select recruiters and follow them to the nectar source;
- The loyalty and recruitment among bees are always a function of the quantity and quality of the food source.

Waggle dance

Foraging of honey bees

(PceliceSaVirtuelnomKamerom.swf)

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- Artificial bees are divided into two groups:
    1. recruiters;
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- Probabilities and roulette wheel are used to handle loyalty and recruitment.

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- Parameters:
  1. B - number of bees;
  2. NC - number of moves during one forward pass.

Bee Colony Optimization - pseudocode

Initialization: Read problem data, parameter values (B and NC),
and stopping criterion.

Do

(1) Assign a(n) (empty) solution to each bee.

(2) *For* $(i = 0; i < NC; i++)$

//**forward pass**

(a) *For* $(b = 0; b < B; b++)$

For $(s = 0; s < f(NC); s++)$//count moves

(i) Evaluate possible moves;

(ii) Choose one move using the roulette wheel;

//**backward pass**

(b) *For* $(b = 0; b < B; b++)$

Evaluate the (partial/complete) solution of bee b;

(c) *For* $(b = 0; b < B; b++)$

Loyalty decision for bee b;

(d) *For* $(b = 0; b < B; b++)$

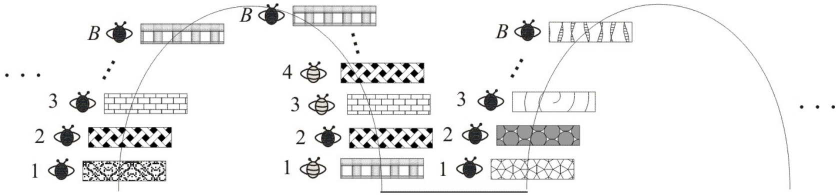
If (b is uncommitted), choose a recruiter by the roulette wheel.

(3) Evaluate all solutions and find the best one. Update x_{best} and $f(x_{best})$

while stopping criterion is not satisfied.

return $(x_{best}, f(x_{best}))$

Bee Colony Optimization - illustration



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- Components/transformations with better characteristics have higher chances to be chosen.

BCO - Backward pass

- Evaluation (Normalization - min.)

$$O_b = \frac{y_{max} - y_b}{y_{max} - y_{min}}, \qquad O_b \in [0,1], \qquad b = 1, 2, \ldots, B$$

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- Recruitment:

$$p_b = \frac{O_b}{\sum_{k=1}^{R} O_k}, \qquad b = 1, 2, \ldots, R$$

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Theoretical verification

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- Constructive variant is considered in details;
- Necessary and sufficient conditions are identified;
- Learning rate is established.

Scheduling independent tasks to identical machines

$T = \{1, 2, \ldots, n\}$ - set of independent tasks,
$M = \{1, 2, \ldots, m\}$ - set of identical machines,
l_i - processing time of task i ($i = 1, 2, \ldots, n$).

Objective: Minimization of completion time of all tasks (*makespan*).

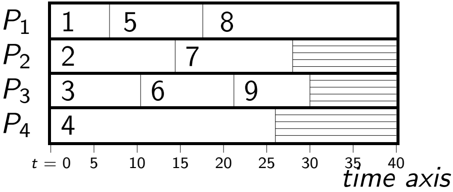


Figure: Gantt diagram–schedule of tasks to processors

Scheduling - mathematical formulation

In order to present a mathematical programming formulation of the problem, let us introduce the binary variables x_{ij} defined in the following way:

$$x_{ij} = \begin{cases} 1, & \text{if task } i \text{ is assigned to processor } j, \\ 0, & \text{otherwise.} \end{cases}$$

The considered scheduling problem is formulated in the following way:

$$\min y \tag{1}$$

$$s.t. \quad \sum_{j=1}^{m} x_{ij} = 1, \;\; 1 \le i \le n, \tag{2}$$

$$y - \sum_{i=1}^{n} l_i x_{ij} \ge 0, \;\; 1 \le j \le m, \tag{3}$$

$$x_{ij} \in \{0, 1\}, \;\; 1 \le i \le n, \;\; 1 \le j \le m, \tag{4}$$

BCO - steps

Construction of solutions: NC tasks are added to the current solution in each forward pass.
Probability of choosing task i equals:

$$p_i = \frac{l_i}{\sum_{k=1}^{K} l_k}, \quad i = 1, 2, \ldots, n \tag{5}$$

with l_i representing the processing time of the i-th task and K being the number of "available" tasks (not previously chosen).
Corresponding processor is selected by a **best fit** rule in such a way that the new solution is not worse than the **current global best** - greedy concept.

The p-Center Problem

- Given is a set of n nodes (locations, customers);
- $D = [d_{ij}]_{n \times n}$ matrix of Euclidean distances between nodes i and j;
- The goal is to locate p facilities (centers) in such a way to minimize the maximum of the distances from each customer to its nearest facility;
- Facilities could be located at any of the given n nodes;
- Customer is assigned to the nearest located facility.

Integer linear program

Binary variables:

$$x_{ij} = \begin{cases} 1, & \text{if user from node } i \text{ is assigned to facility located at node } j, \\ 0, & \text{otherwise.} \end{cases}$$

$$y_j = \begin{cases} 1, & \text{if facility is located at node } j, \\ 0, & \text{otherwise.} \end{cases}$$

The objective to minimize maximum distance between customer and the corresponding facility can be given as

$$\min \max \sum_{j=1}^{n} d_{ij} x_{ij}$$

Integer linear program (Constraints)

$$\sum_{j=1}^{n} x_{ij} = 1, \quad 1 \le i \le n, \tag{6}$$

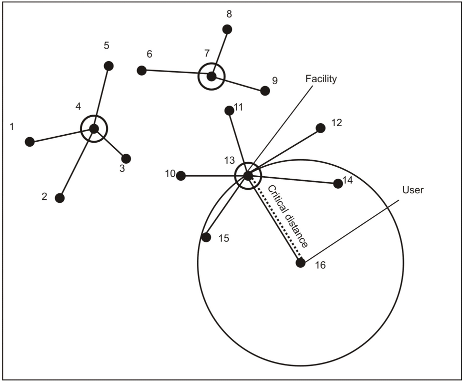
$$x_{ij} \le y_j, \quad 1 \le i \le n, \quad 1 \le j \le n, \tag{7}$$

$$\sum_{j=1}^{n} y_j = p, \tag{8}$$

$$z - \sum_{j=1}^{n} d_{ij} x_{ij} \ge 0, \quad 1 \le i \le n, \tag{9}$$

$$x_{ij}, y_j \in \{0, 1\}, \quad 1 \le i \le n, \quad 1 \le j \le n. \tag{10}$$

Solution example



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- Q locations are removed from the center list in a greedy manner;
- $Q \in [0, p]$ if $5 \cdot p < n$, otherwise $Q \in [0, \frac{n-2.5\cdot p}{2.5}]$.

Summary and classification

- Routing: the traveling salesman problem, vehicle routing problem, vehicle routing problem with time windows, Vehicle rerouting in the case of unexpectedly high demand in distribution systems, routing and wavelength assignment (RWA) in all-optical networks;

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- Location: the p-median problem, traffic sensors locations problem on highways, inspection stations locations in transport networks, anti-covering location problem, p-center problem, location of distributed generation resources, and capacitated plant location problem;
- Scheduling: static scheduling of independent tasks on homogeneous multiprocessor systems, scheduling dependent tasks to homogeneous systems, open-shop scheduling, the ride-matching problem, job shop scheduling, task scheduling in computational grids, backup allocation problem, and berth allocation problem;

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- Selection: feature selection problem.

PhD thesis

[1] M. Šelmić, *Location problems on transport networks by computational intelligence methods*, PhD thesis, Faculty of Traffic and Transportation, University of Beograde, 2011.

[2] M. Nikolić, *Resolving the consequences of traffic disturbances by bee colony optimization*, PhD thesis, Faculty of Traffic and Transportation, University of Beograde, 2015.

[3] T. Stojanović, *The development and analisys of metaheuristics for satisfiability in probabilistic logics*, Faculty of Science, University of Kragujevac, 2015.

[4] T. Jakšić Krüger, *The development, parallelization and theoretical verification of bee colony optimization*, Faculty of Technical Sciences, University of Novi Sad, 2015.

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- Advanced parallelization;
- New applications.

Thank you for the attention!

Questions?

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