

VII Simpozijum MATEMATIKA I PRIMENE,  
Matematički fakultet, Univerzitet u Beogradu, 2016

*Uticaj sile Jarkovskog i rezonanci u  
srednjem kretanju na kretanje asteroida u  
Glavnom asteroidnom pojasu*

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# *Uvod*

- ♦ Gravitacioni i negravitacioni mehanizmi  
Rezonance u srednjem kretanju (RSK)  
Sila Jarkovskog
- ♦ Funkcionalna veza između snage rezonance ( $SR$ ), sile Jarkovskog ( $da/dt$ ) i vremena ( $dtr$ )  
(Milić Žitnik i Novaković, 2016)
- ♦ Proširena analiza i novi rezultati  
(Milić Žitnik, 2016)

# *Metode*

- ♦ Ispitivanje interakcije između RSK i sile Jarkovskog  
Milić Žitnik i Novaković (2015, 2016)
- ♦ Generisanje orbitalnih elemenata za 66000 test asteroida
- ♦ Integrator ORBIT9 (Milani i Nobili, 1988)
- ♦ Dva dinamička modela
- ♦ 10 različitih vrednosti sile Jarkovskog  
{ $-4 \times 10^{-5}$ , ...,  $-2 \times 10^{-3}$  AU/Myr}
- ♦ 11 izolovanih RSK sa Jupiterom

- ♦ Vreme koje asteroid provede u rezonanci:

$$dtr = \Delta t - \Delta a / yark$$

$$\Delta t = t_2 - t_1$$

$$\Delta a = a_2 - a_1$$

$$yark = da / dt$$

- Asimetrična Laplasova statistička raspodela

(Đorić et al., 2007)

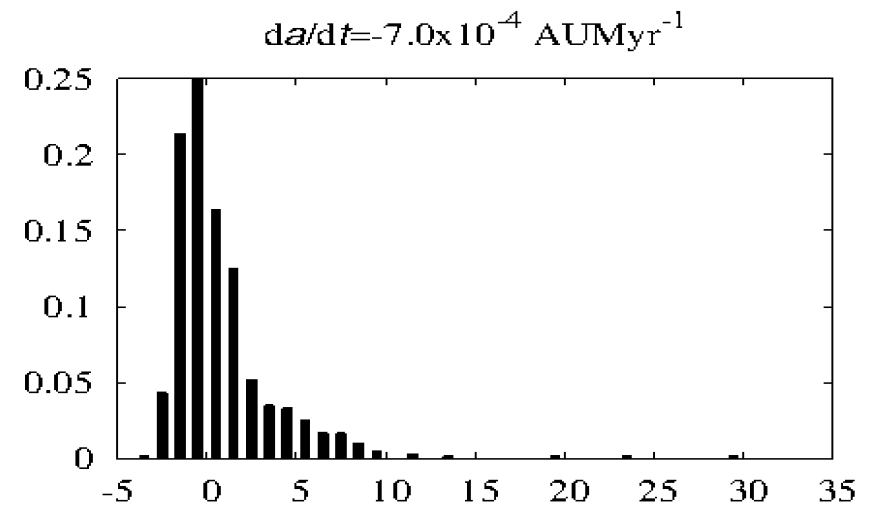
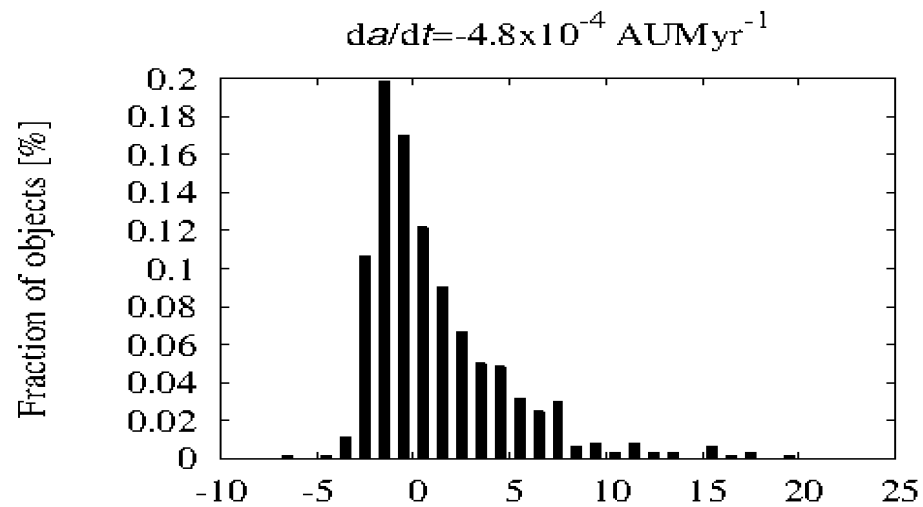
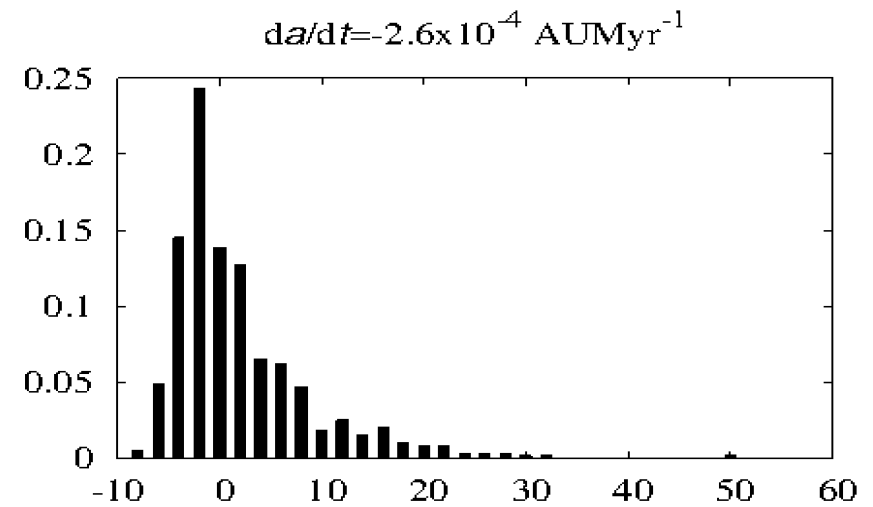
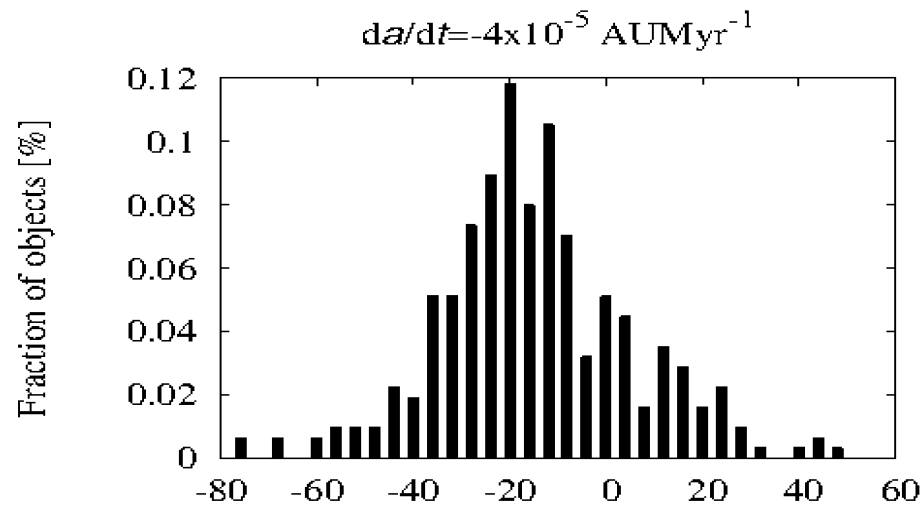
$$g(x) = (1-p) / l \exp(- |x-a| / l), x \leq a$$

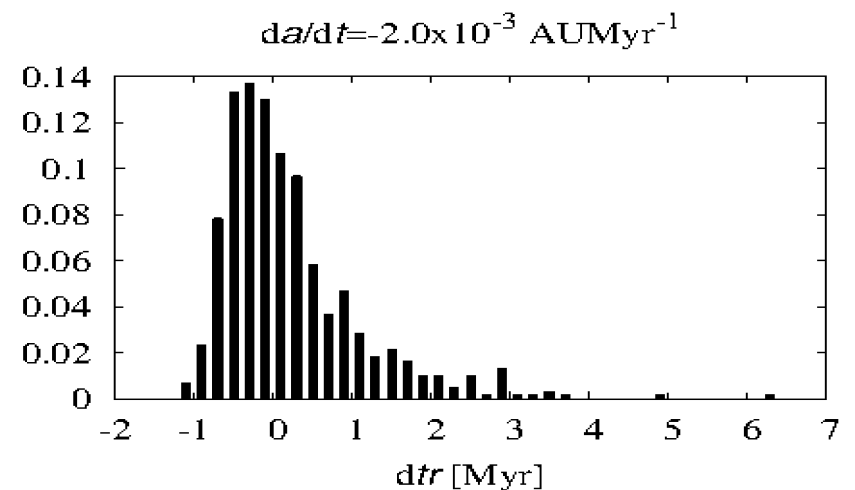
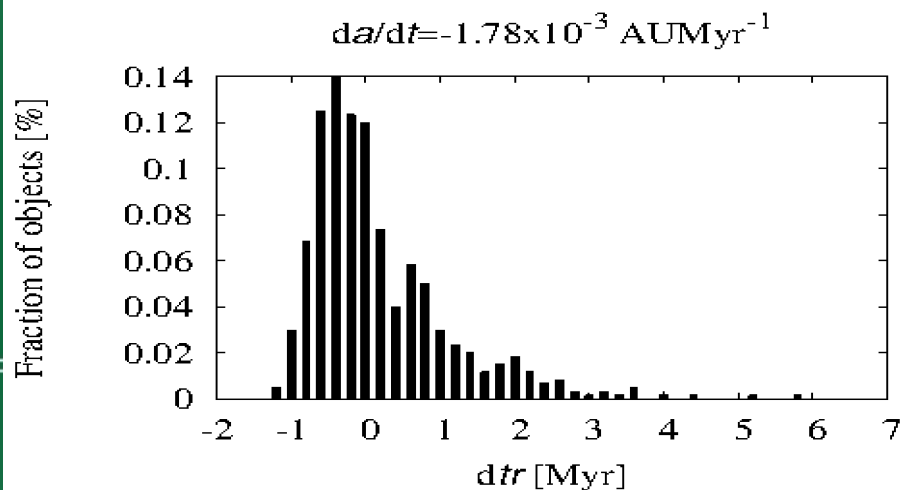
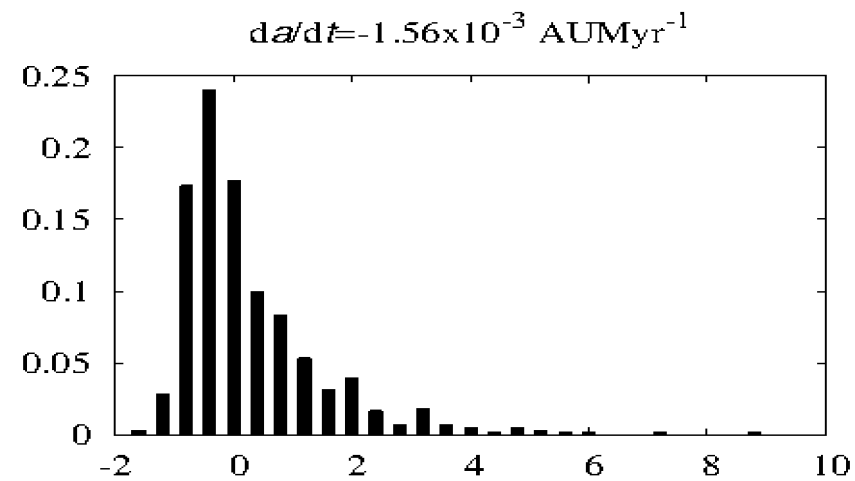
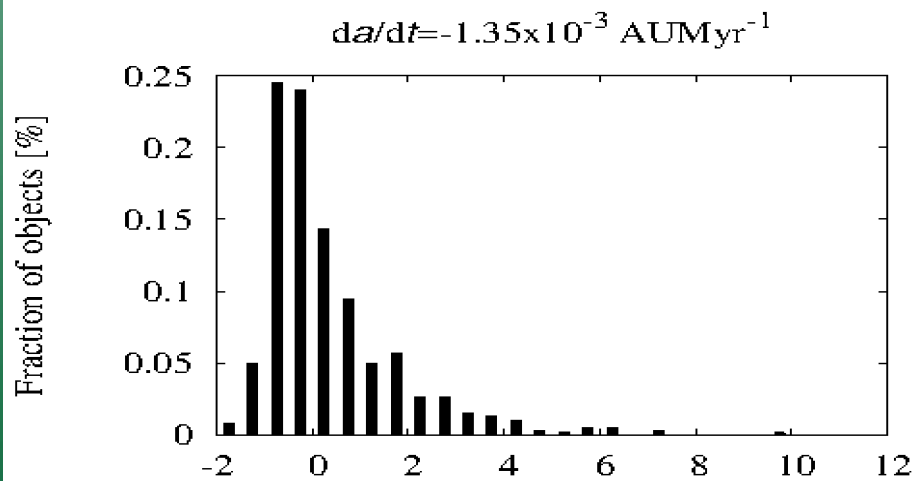
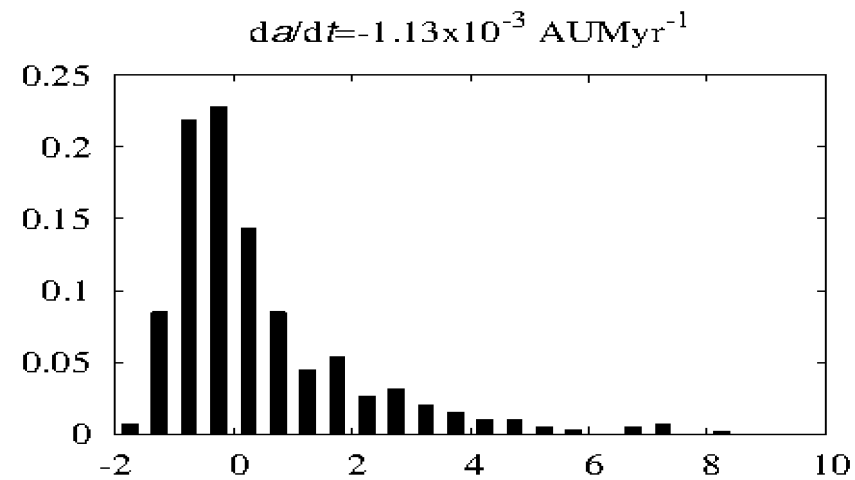
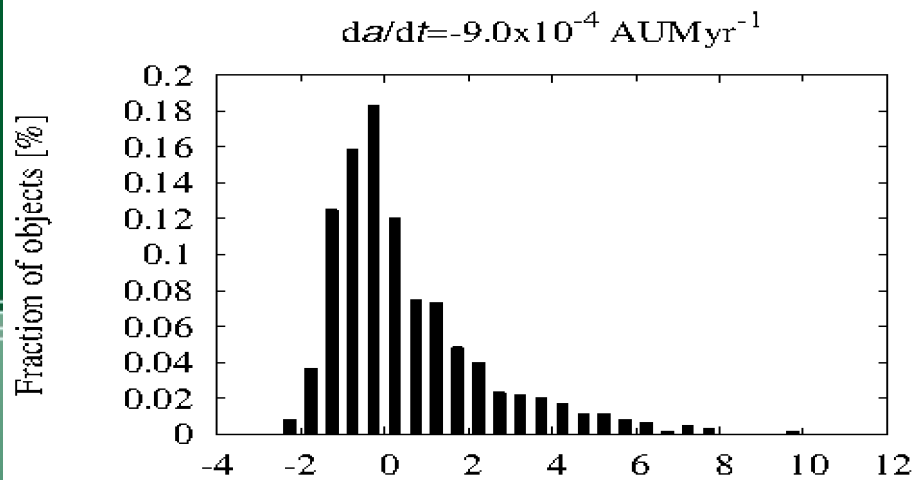
$$g(x) = p / l \exp(- |x-a| / l), x > a$$

$a$  – parametar lokacije,  $l > 0$ ,  $0 < p < 1$

# Rezultati

## ◆ Raspodela $dtr$ za test asteroide u 9:4 rezonanci





- ◆ Kolmogorov-Smirnov test:

Testirana je nulta hipoteza  $H_0: P_0 = P_1$  sa nivoom značajnosti  $\alpha = 0.05$  i  $\alpha = 0.01$  nad  $\sim 31\%$  histograma za različite rezonance i vrednosti efekta Jarkovskog.

Za  $\alpha = 0.05$   $H_0$  je prihvatljivo za 20 parova histograma (od ispitivanih 26 parova).

Za  $\alpha = 0.01$   $H_0$  je prihvatljivo za 21 par histograma (od ispitivanih 26 parova).

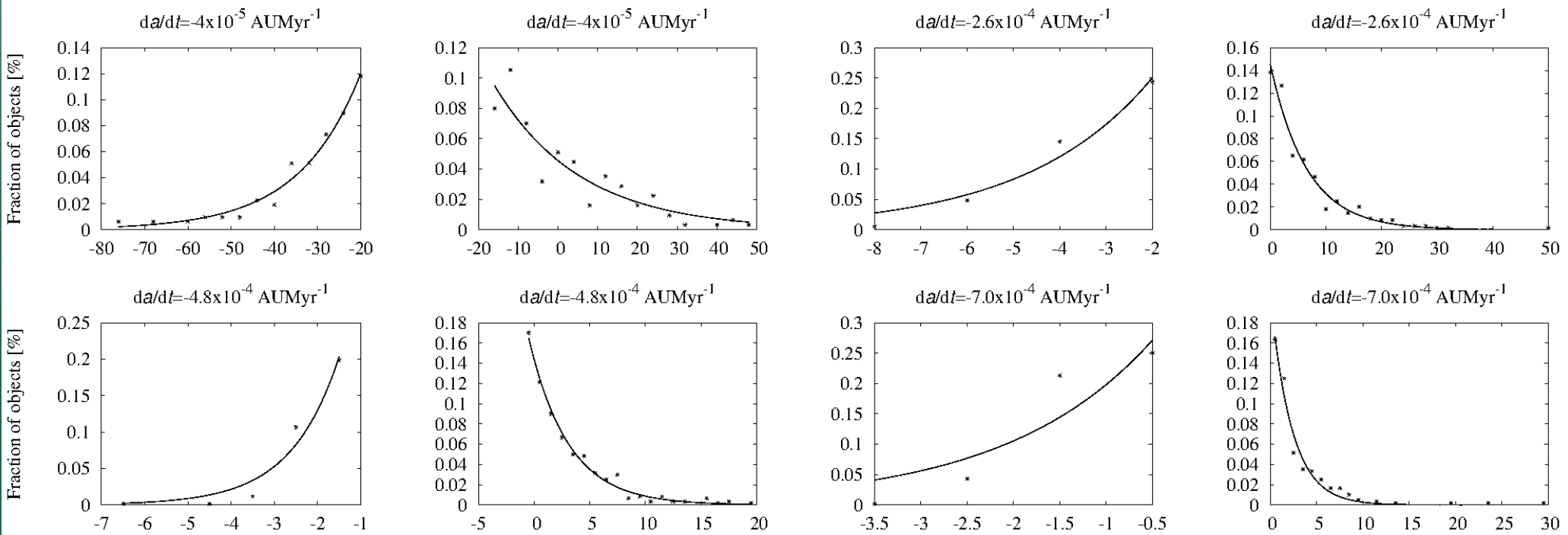
=> Podaci za *dtr* prikazani na histogramima pripadaju istoj raspodeli ili sličnim raspodelama.

# ♦ Laplasova modifikovana asimetrična raspodela (Milić Žitnik, 2016):

$$g(x) = (1-p_l)/l_l \exp(-|x-a|/l_l), x \leq a$$

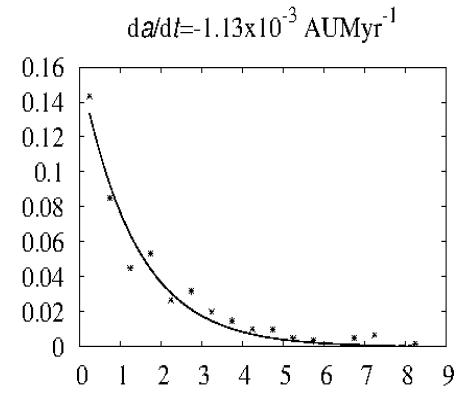
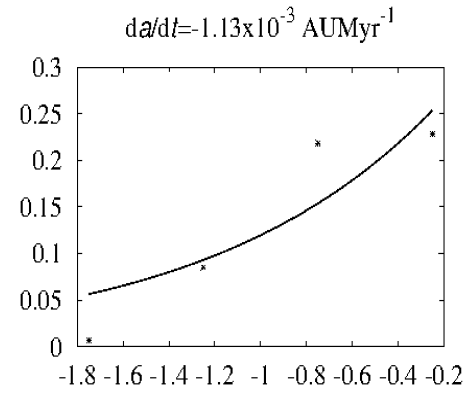
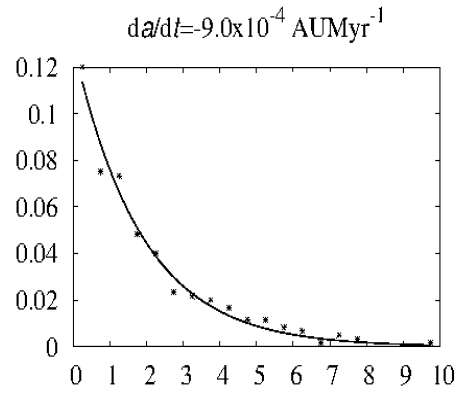
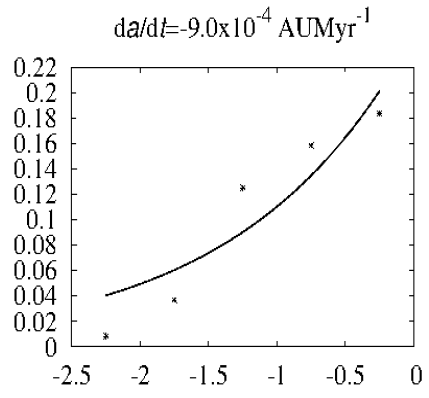
$$g(x) = p_r/l_r \exp(-|x-a|/l_r), x > a$$

$a$  – parametar lokacije,  $\{l_l, l_r\} > 0$ ,  $0 < \{p_l, p_r\} < 1$

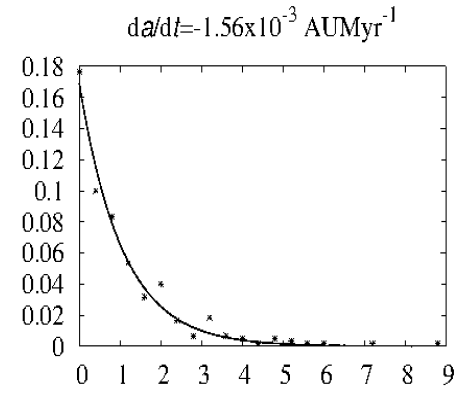
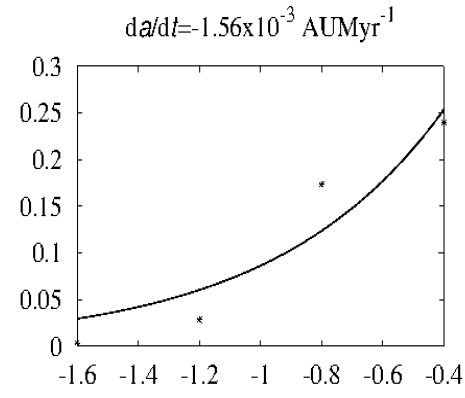
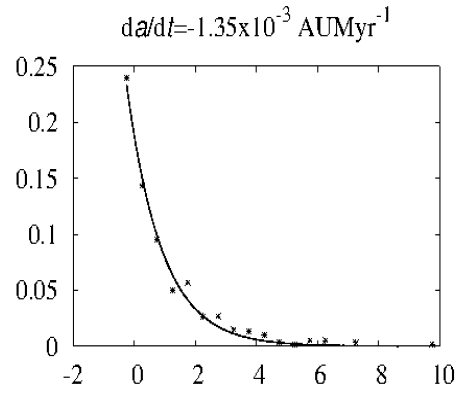
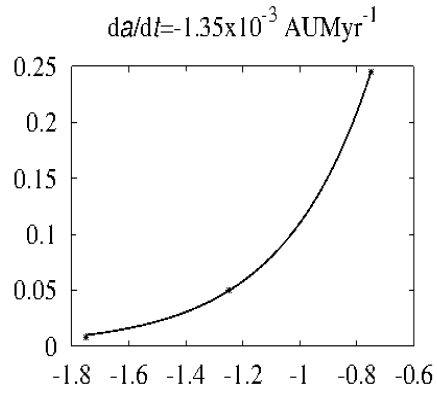




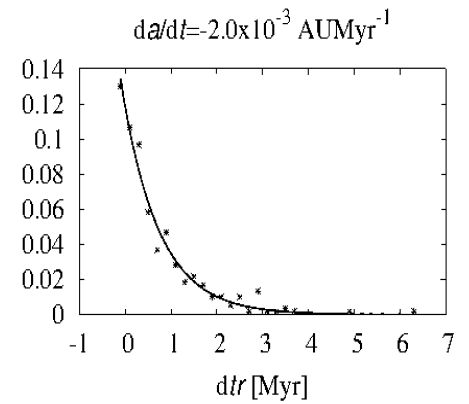
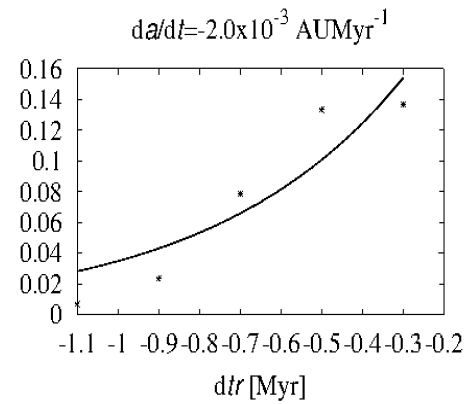
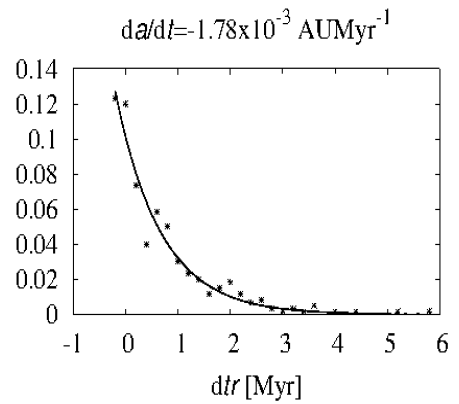
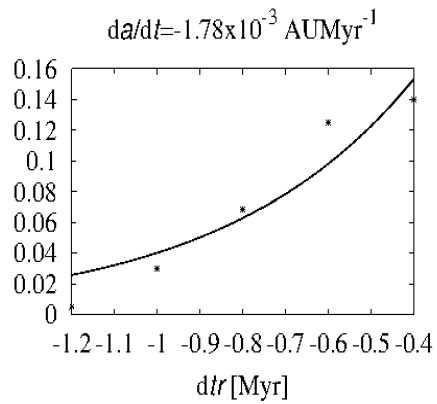
Fraction of objects [%]



Fraction of objects [%]



Fraction of objects [%]



$dtr$  [Myr]

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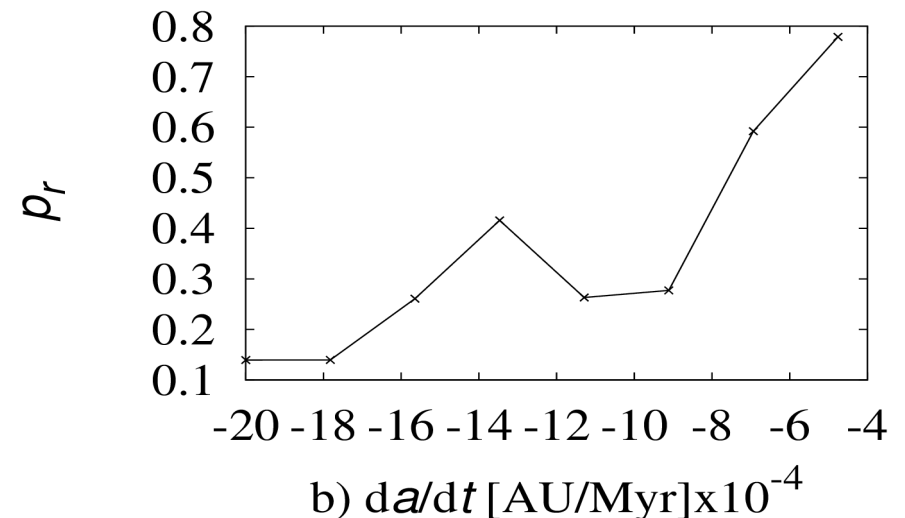
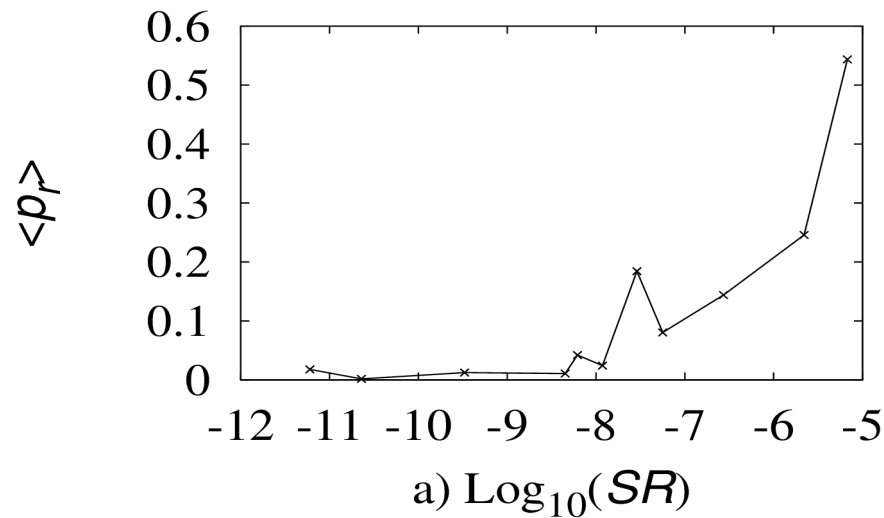
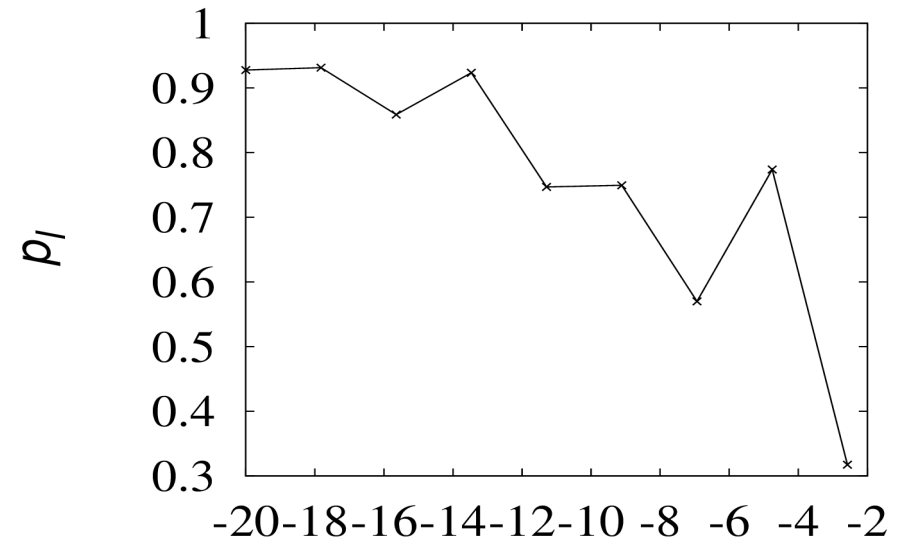
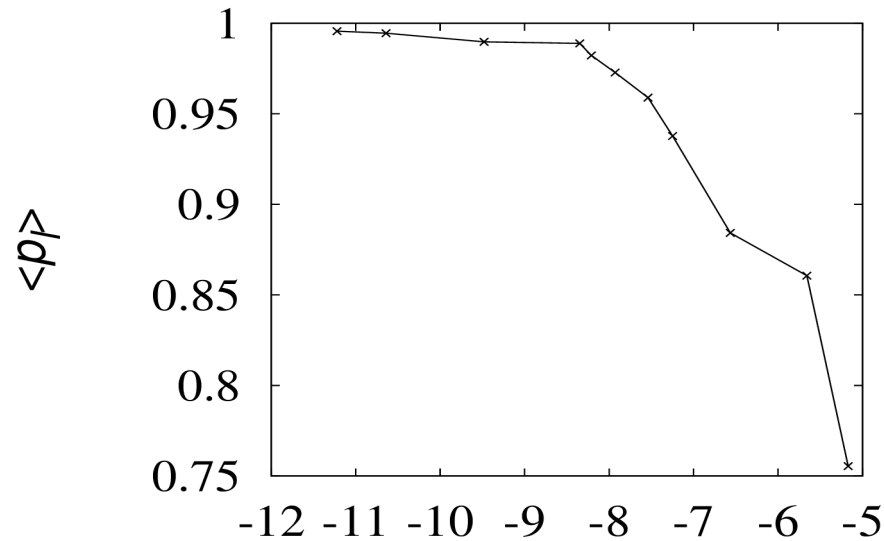
## ♦ Pirsonov ( $\chi^2$ ) test

Testirana je nulta hipoteza  $H_0$ : “Podaci za *dtr* imaju modifikovanu Laplasovu asimetričnu raspodelu” sa nivoom značajnosti  $\alpha=0.05$ .

Test je prošlo 80% podataka.

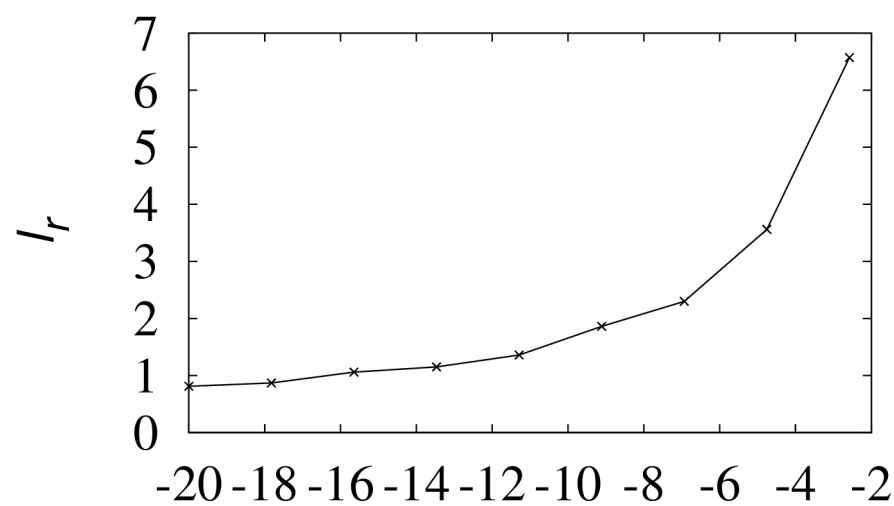
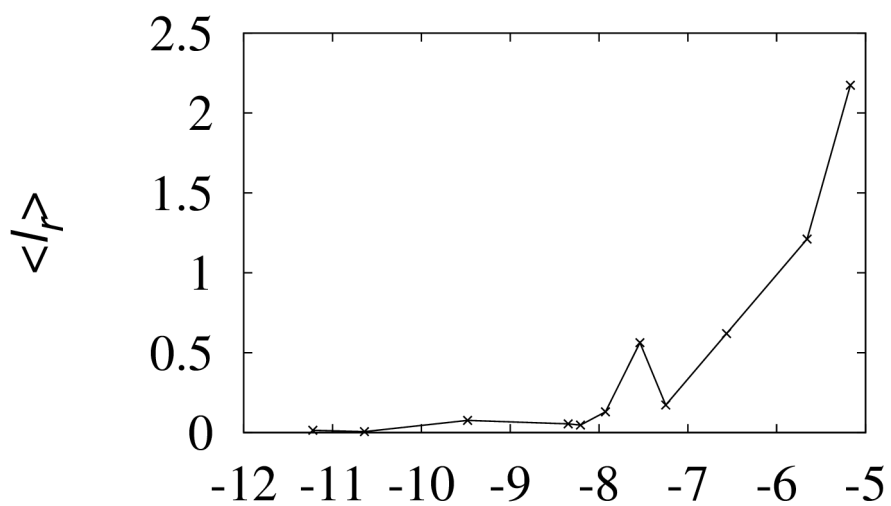
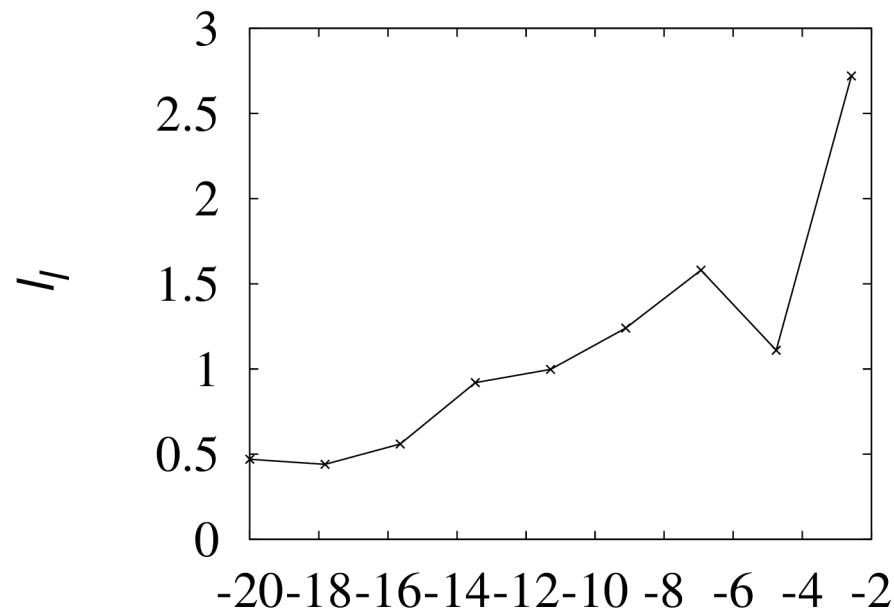
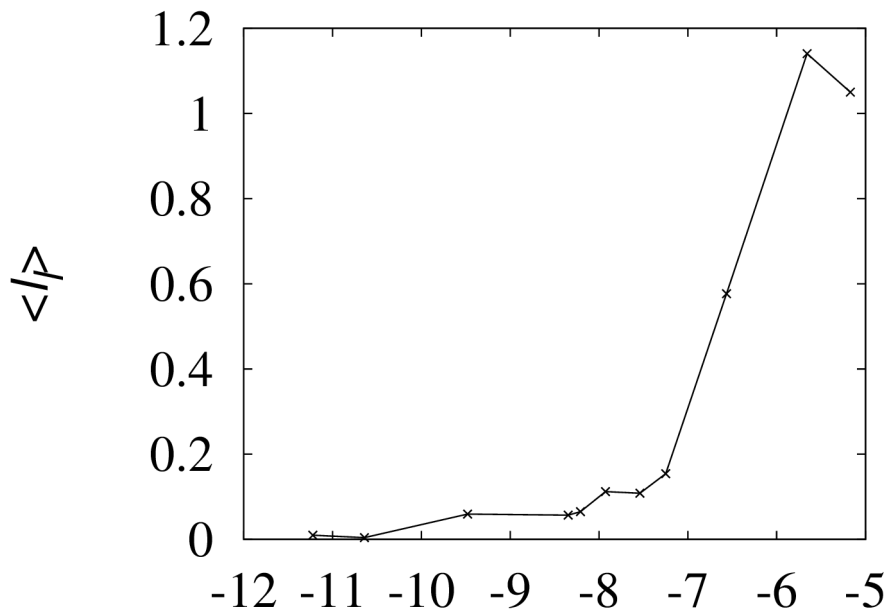
Velika disperzija kod preostalih 20% podataka.

# Analiza parametara modifikovane Laplasove asimetrične raspodele



a)  $\text{Log}_{10}(SR)$

b)  $da/dt [\text{AU/Myr}] \times 10^{-4}$



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b)  $da/dt [\text{AU/Myr}] \times 10^{-4}$

◆ Funkcionalna veza između  $\{l_l, l_r, p_l, p_r\}$ ,  $da/dt$  i  $SR$

$$\log_{10}(\{l_l, l_r, p_l, p_r\}) = a \log_{10}(SR) + b \log_{10}(da/dt) + c$$

	$a \pm \sigma_a$	$b \pm \sigma_b$	$c \pm \sigma_c$
$l_l$	$0.397 \pm 0.014$	$-0.919 \pm 0.055$	$-0.980 \pm 0.205$
$l_r$	$0.434 \pm 0.019$	$-0.928 \pm 0.077$	$-0.619 \pm 0.284$
$p_l$	$-0.018 \pm 0.003$	$0.069 \pm 0.012$	$0.035 \pm 0.044$
$p_r$	$0.426 \pm 0.024$	$-0.926 \pm 0.094$	$-1.206 \pm 0.344$

♦ Jednačina veze između  $\langle dtr \rangle$ ,  $SR$ ,  $da/dt$ ,  $e$

$$\langle dtr \rangle = c_1 (SR)^\beta (da/dt)^\gamma / \log_{10}$$

$$\log_{10}(\langle dtr \rangle) = \beta \log_{10}(SR) + \gamma \log_{10}(da/dt) + c_2$$

Za  $e \sim 0.1$ :

$$\beta = 0.44 \pm 0.03, \gamma = -1.09 \pm 0.20, c_2 = 4.35 \pm 0.66 \text{ za 11 RSK}$$

$$\beta = 0.47 \pm 0.04, \gamma = -0.97 \pm 0.15, c_2 = 5.11 \pm 0.54 \text{ za 6 RSK}$$

Za  $0.025 \leq e \leq 0.4$ :

(Tabela 3 iz Milić Žitnik, 2016)

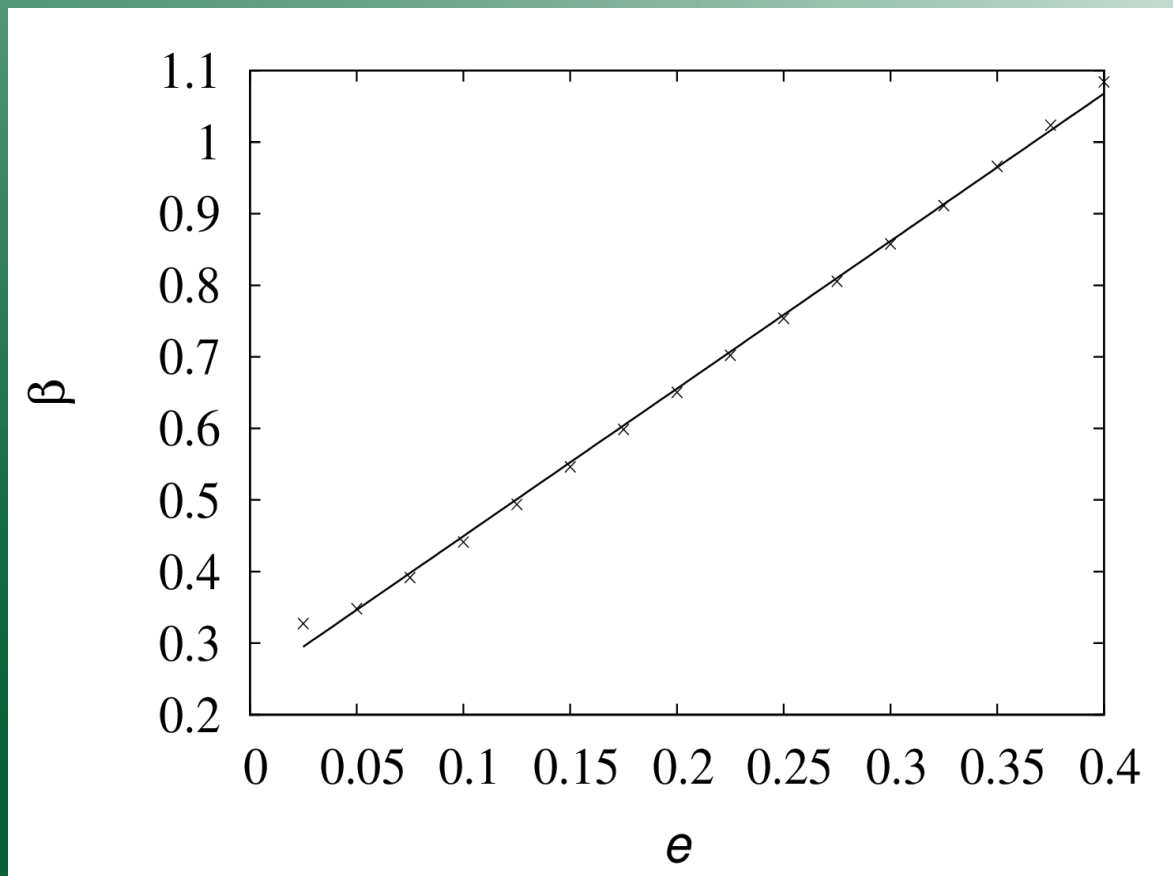
$\Rightarrow \beta$  raste sa  $e$  (za sve RSK)

$\gamma = -1.092$  (za sve vrednosti  $e$ )

$c_2$  raste sa  $e$  (osim za  $e=0.025$ )

♦ Zavisnost između  $e$  i  $\beta$

$$\beta = ae + b$$



$$a = 2.06 \pm 0.02, b = 0.24 \pm 0.01$$

# Zaključak

- ♦ Jednačina koja važi za  $0.025 \leq e \leq 0.4$  :

$$\log_{10}(\langle dtr \rangle) = (2.06e + 0.24)\log_{10}(SR) - 1.09\log_{10}(da/dt) + c_2 .$$

- ♦ Modifikovana Laplasova asimetrična raspodela može poslužiti za generisanje  $dtr$  za određeni broj objekata sa poznatim vrednostima brzine Jarkovskog u RSK sa poznatom jačinom.
- ♦ Ovi rezultati mogu biti implementirani u različite Monte-Karlo metode za kretanje asteroida preko RSK u glavnom asteroidnom pojasu.
- ♦ Trenutno radimo na simulaciji kretanja asteroida preko rezonance 17:8 sa novim vrednostima sile Jarkovskog.



***Hvala vam na pažnji!***