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On the physical foundation of equipartition in supernova remnants

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Equipartition (eqp) calculation

- The eqp or minimum-energy calculation
- Determination of the magnetic field strength and minimal energy contained in the magnetic field and cosmic ray particles
- Directly from radio-continuum observations – synchrotron emission

Historical review

- Pacholczyk (1970) – integration of the cosmic ray energy spectrum over frequencies – “classical equipartition”
- Revised equipartition – Beck & Krause (2005) – integration of the cosmic ray energy spectrum over energies

Assumptions for our derivation

- Bell's (1978) injection $E_{\rm inj} \approx 4\ 1/2 m_{\rm p} v_{\rm s}^2$
- Shock velocity of an SNR is low enough ($v_{\rm s}$ << 7000 km/s – older SNRs) =>

$$E_{\rm inj} << m_{\rm e}c^2 \quad (p_{\rm inj}^{\rm e} << m_{\rm e}c)$$

- Particles are injected into the acceleration process all with the same injection energy
- Plasma is fully ionized and globally electroneutral

Derivation

- Assuming power law momentum distribution ($N = kp^{-\gamma}$), the energy density of one cosmic ray "ingredient" is:

$$
\begin{aligned}
\epsilon &= \int_{p_{\mathrm{inj}}}^{p_\infty} 4\pi k p^{-\gamma}(\sqrt{p^2c^2+m^2c^4}-mc^2)\mathrm{d}p \\
&\approx \int_0^\infty 4\pi k p^{-\gamma}(\sqrt{p^2c^2+m^2c^4}-mc^2)\mathrm{d}p \\
&= 4\pi kc(mc)^{2-\gamma}\int_0^\infty x^{-\gamma}(\sqrt{x^2+1}-1)\mathrm{d}x, \quad x=\frac{p}{mc} \\
&= K(mc^2)^{2-\gamma}\frac{\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})}{2\sqrt{\pi}(\gamma-1)}.
\end{aligned}
$$

- K is from $N = KE^{-\gamma}$; analytical solutions only for $2 < \gamma < 3$ (the spectral indices of SNRs: $0.5 < \alpha < 1$)

- Total CR energy density (for all ingredients: electrons, protons, heavier ions):

$$\begin{aligned}
\epsilon_{\rm CR} &= \frac{\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})}{2\sqrt{\pi}(\gamma-1)}\left(K_e(m_e c^2)^{2-\gamma} + \sum_i K_i(m_i c^2)^{2-\gamma}\right) \\
&= \frac{\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})}{2\sqrt{\pi}(\gamma-1)}\left(K_e(m_e c^2)^{2-\gamma} + K_p(m_p c^2)^{2-\gamma}\sum_i \frac{n_i}{n_p}\left(\frac{m_i}{m_p}\right)^{(3-\gamma)/2}\right) \\
&= \frac{\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})}{2\sqrt{\pi}(\gamma-1)}K_e(m_e c^2)^{2-\gamma}\left(1 + \frac{n}{n_e}\left(\frac{m_p}{m_e}\right)^{(3-\gamma)/2}\sum_i \frac{n_i}{n}\left(\frac{m_i}{m_p}\right)^{(3-\gamma)/2}\right) \\
&= K_e(m_e c^2)^{2-\gamma}\frac{\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})}{2\sqrt{\pi}(\gamma-1)}(1+\kappa), \qquad (2)
\end{aligned}$$

$$\kappa = \left(\frac{m_p}{m_e}\right)^{(3-\gamma)/2} \frac{\sum_i A_i^{(3-\gamma)/2} \nu_i}{\sum_i Z_i \nu_i}$$

where κ is the energy ratio, ν_i are the ion abundances, A_i and Z_i are masses and charge numbers of elements; we neglected energy losses.

- The synchrotron emissivity:

$$\varepsilon_\nu = c_5 K_e (B \sin\Theta)^{(\gamma+1)/2} \left(\frac{\nu}{2c_1}\right)^{(1-\gamma)/2}$$

- The flux density:

$$S_\nu = \frac{L_\nu}{4\pi d^2} = \frac{\frac{4\pi}{3} R^3 f \mathcal{E}_\nu}{4\pi d^2} = \frac{4\pi}{3} \varepsilon_\nu f \theta^3 d$$

- The isotropic distribution for the orientation of pitch angles (radial magnetic field) (Longair 1994) :

$$\frac{1}{2}\int_0^\pi (\sin\Theta)^{(\gamma+3)/2} \mathrm{d}\Theta = \frac{\sqrt{\pi}}{2} \frac{\Gamma(\frac{\gamma+5}{4})}{\Gamma(\frac{\gamma+7}{4})}$$

- For the total energy we have:

$$E = \frac{4\pi}{3} R^3 f(\epsilon_{\rm CR} + \epsilon_B), \quad \epsilon_B = \frac{1}{8\pi} B^2,$$

$$E = \frac{4\pi}{3} R^3 f \left(K_e (m_e c^2)^{2-\gamma} \frac{\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})}{2\sqrt{\pi}(\gamma-1)} (1+\kappa) + \frac{1}{8\pi} B^2 \right)$$

- Looking for the minimum energy with respect to *B*, d*E*/d*B*=0:

$$\frac{\mathrm{d}K_e}{\mathrm{d}B} (m_e c^2)^{2-\gamma} \frac{\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})}{2\sqrt{\pi}(\gamma-1)} (1+\kappa) + \frac{1}{4\pi} B = 0$$

- Using eqs. for synchrotron emissivity, flux density and distribution of magnetic field lines:

$$\frac{\mathrm{d}K_e}{\mathrm{d}B} = -\frac{3}{4\pi}\frac{S_\nu}{f\theta^3 d}\frac{1}{c_5}\Big(\frac{\nu}{2c_1}\Big)^{-(1-\gamma)/2}\frac{(\gamma+1)\Gamma(\frac{\gamma+7}{4})}{\sqrt{\pi}\Gamma(\frac{\gamma+5}{4})}B^{-(\gamma+3)/2}$$

- Finally:

$$B = \Big(\frac{3}{2\pi}\frac{(\gamma+1)\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})\Gamma(\frac{\gamma+7}{4})}{(\gamma-1)\Gamma(\frac{\gamma+5}{4})}\frac{S_\nu}{fd\theta^3}\cdot$$

$$\cdot(m_e c^2)^{2-\gamma}\frac{(2c_1)^{(1-\gamma)/2}}{c_5}(1+\kappa)\nu^{(\gamma-1)/2}\Big)^{2/(\gamma+5)}$$

- For easier calculation:

$$B\ [\mathrm{Ga}] \approx \Bigg(6.286 \cdot 10^{(9\gamma-79)/2} \frac{\gamma+1}{\gamma-1} \frac{\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})\Gamma(\frac{\gamma+7}{4})}{\Gamma(\frac{\gamma+5}{4})} (m_e c^2)^{2-\gamma} \cdot \tag{12}$$

$$\cdot \quad \frac{(2c_1)^{(1-\gamma)/2}}{c_5} (1+\kappa) \frac{S_\nu[\mathrm{Jy}]}{f\ d[\mathrm{kpc}]\ \theta[\mathrm{arcmin}]^3} \nu[\mathrm{GHz}]^{(\gamma-1)/2}\Bigg)^{2/(\gamma+5)},$$

- We also have: $E_B = \frac{\gamma+1}{4} E_{\mathrm{CR}}, \quad E_{\min} = \frac{\gamma+5}{\gamma+1} E_B$

Arbutina et al. (2012, 2013)
http://poincare.matf.bg.ac.rs/~arbo/eqp

Equipartition calculation for supernova remnants

If you are using this calculator, please cite: B. Arbutina, D. Urošević, M. M. Andjelić, M. Z. Pavlović and B. Vukotić, "Modified equipartition calculation for supernova remnants", 2012, *Astrophys. J.*, **746**, 79 (arXiv:1111.5465). See the above paper for the explanation what this programme does. For more information contact: arbo@math.rs.

Spectral index, α = 0.5 (0.5-1)

Frequency, ν = 1 [GHz]

Flux density, S_ν = 10 [Jy]

Distance, d = 1 [kpc]

Angular radius, θ = 10 [arcmin]

Filling factor, f = 0.25 (0-1)

Shock velocity, v_s = 0 [km/s] (if unknown leave 0)

$_{3}\mathrm{Li}^{7}$	$_{4}\mathrm{Be}^{9}$	$_{5}\mathrm{B}^{10/11}$	$_{6}\mathrm{C}^{12}$	$_{7}\mathrm{N}^{14}$	$_{8}\mathrm{O}^{16}$	$_{9}\mathrm{F}^{19}$
0	0	0 0	0	0	0	0

$_{11}\mathrm{Na}^{23}$	$_{12}\mathrm{Mg}^{24\text{-}26}$	$_{13}\mathrm{Al}^{27}$	$_{14}\mathrm{Si}^{28\text{-}30}$	$_{15}\mathrm{P}^{31}$	$_{16}\mathrm{S}^{32\text{-}34/36}$	$_{17}\mathrm{Cl}^{35/37}$
0	0 / 0 0	0	0 / 0 0	0	0 0 / 0 0	0 0

$_{19}\mathrm{K}^{40*}$	$_{20}\mathrm{Ca}^{40/42\text{-}44}$	$_{21}\mathrm{Sc}^{45}$	$_{22}\mathrm{Ti}^{46\text{-}50}$	$_{23}\mathrm{V}^{51}$	$_{24}\mathrm{Cr}^{52\text{-}54}$	$_{25}\mathrm{Mn}^{55}$	$_{26}\mathrm{Fe}^{56\text{-}58}$	$_{27}\mathrm{Co}^{59}$	$_{28}\mathrm{Ni}^{58/60\text{-}62/64}$	$_{29}\mathrm{Cu}^{63/65}$	$_{30}\mathrm{Zn}^{64*}$	$_{31}\mathrm{Ga}^{69/71}$	$_{32}\mathrm{Ge}^{70/72\text{-}74}$	$_{33}\mathrm{As}^{75}$	$_{34}\mathrm{Se}^{74/76\text{-}78/80}$	$_{35}\mathrm{Br}^{79/81}$
0	0 0 / 0 0	0	0 0 / 0 0 / 0	0	0 / 0 0	0	0 / 0 0	0	0 0 / 0 0 0	0 0	0	0 0	0 0 / 0 0	0	0 0 / 0 0 / 0	0 0

$_{37}\mathrm{Rb}^{85}$	$_{38}\mathrm{Sr}^{84/86\text{-}88}$	$_{39}\mathrm{Y}^{89}$	$_{40}\mathrm{Zr}^{90\text{-}92}$	$_{41}\mathrm{Nb}^{93}$	$_{42}\mathrm{Mo}^{92/94\text{-}98}$	$_{43}\mathrm{Tc}^{98*}$	$_{44}\mathrm{Ru}^{96/98\text{-}102/104}$	$_{45}\mathrm{Rh}^{103}$	$_{46}\mathrm{Pd}^{102/104\text{-}106/108/110}$	$_{47}\mathrm{Ag}^{107/109}$	$_{48}\mathrm{Cd}^{110\text{-}112}$	$_{49}\mathrm{In}^{113}$	$_{50}\mathrm{Sn}^{112/114\text{-}120/122/124}$	$_{51}\mathrm{Sb}^{121/123}$	$_{52}\mathrm{Te}^{122/124\text{-}126}$	$_{53}\mathrm{I}^{127}$
0	0 0 / 0 0	0	0 / 0 0	0	0 0 / 0 / 0 0 / 0	0	0 0 0 / 0 0 0 / 0	0	0 0 0 / 0 0 0	0 0	0 / 0 0	0	0 0 / 0 0 / 0 / 0 0 / 0 0	0 0	0 0 / 0 0	0

Warning: Division by zero in /home/arbo/pu

Warning: Division by zero in /home/arbo/pu

Warning: Division by zero in /home/arbo/pu

Warning: Division by zero in /home/arbo/public_html/eqp/index.php on line 262

Warning: Division by zero in /home/arbo/public_html/eqp/index.php on line 262

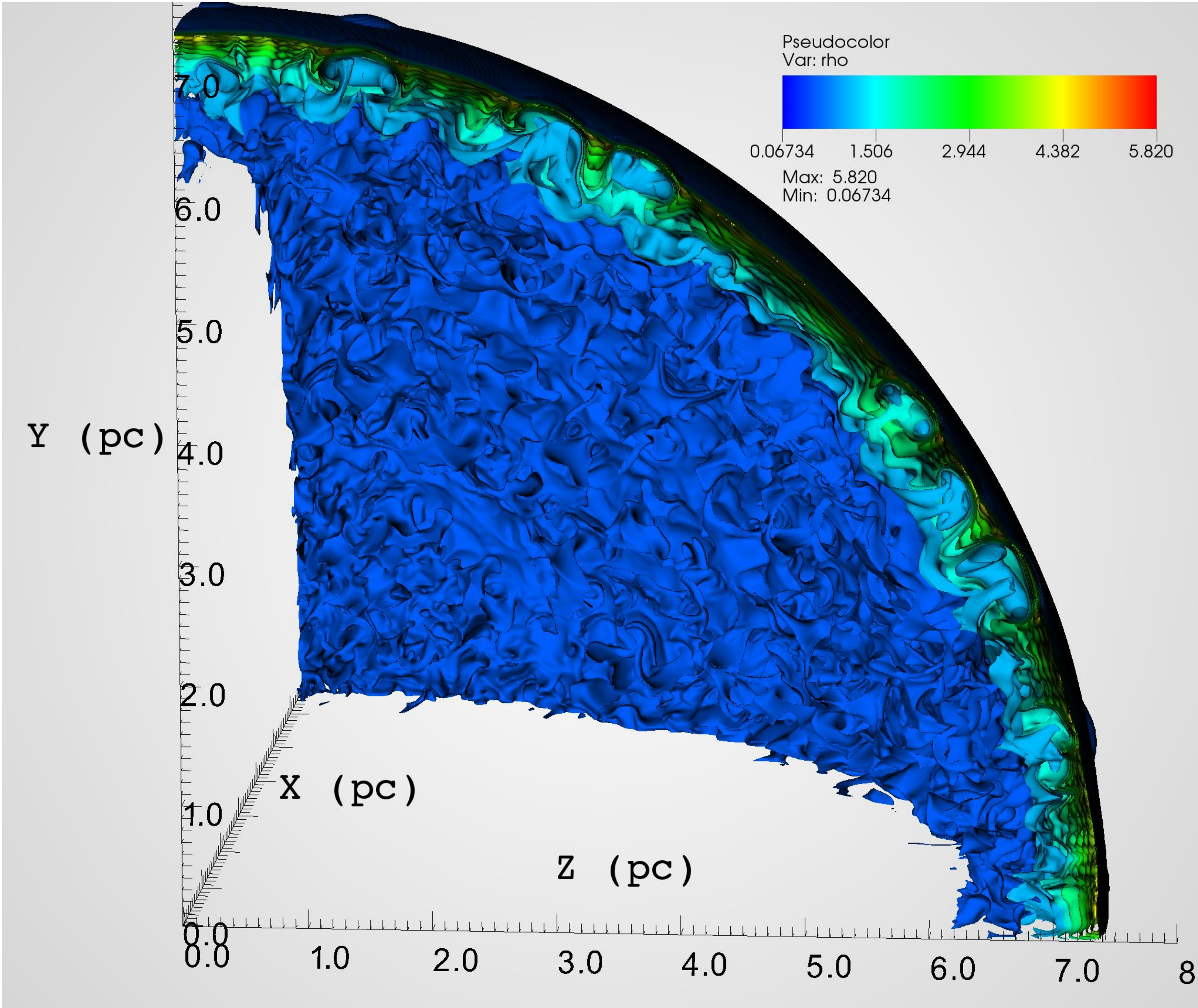
Warning: Division by zero in /home/arbo/public_html/eqp/index.php on line 886

Warning: Division by zero in /home/arbo/public_html/eqp/index.php on line 256
B = 0 μGa

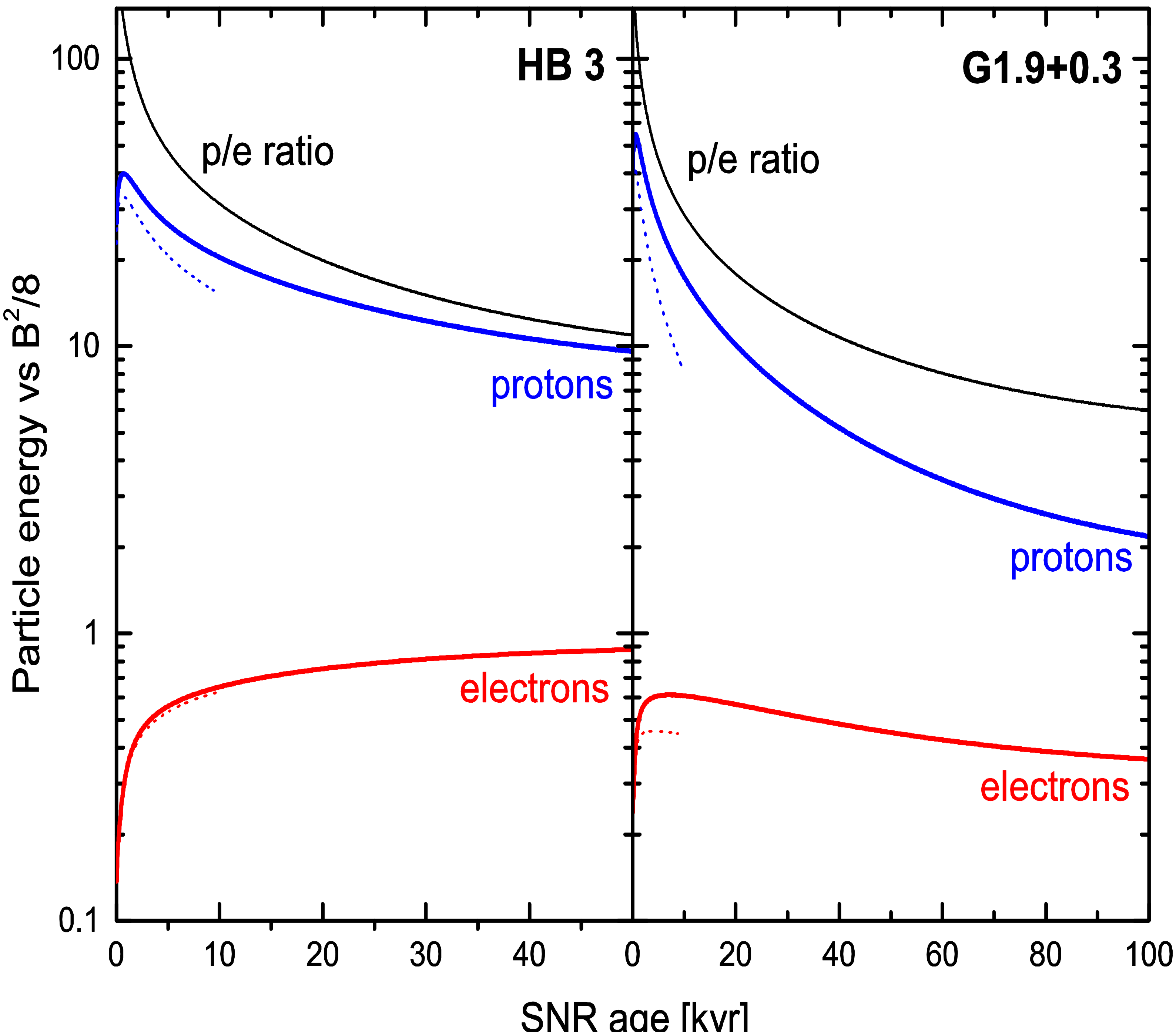
Emin = 0 ergs

Physical foundation of eqp in SNRs

Yes or No?



Pseudocolor
Var: rho
0.06734
1.506
2.944
4.382
5.820
Max: 5.820
Min: 0.06734
Y (pc)
X (pc)
Z (pc)
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.



HB 3
G1.9+0.3
p/e ratio
p/e ratio
protons
protons
electrons
electrons
Particle energy vs B²/8
SNR age [kyr]
100
10
1
0.1
0
10
20
30
40
0
20
40
60
80
100

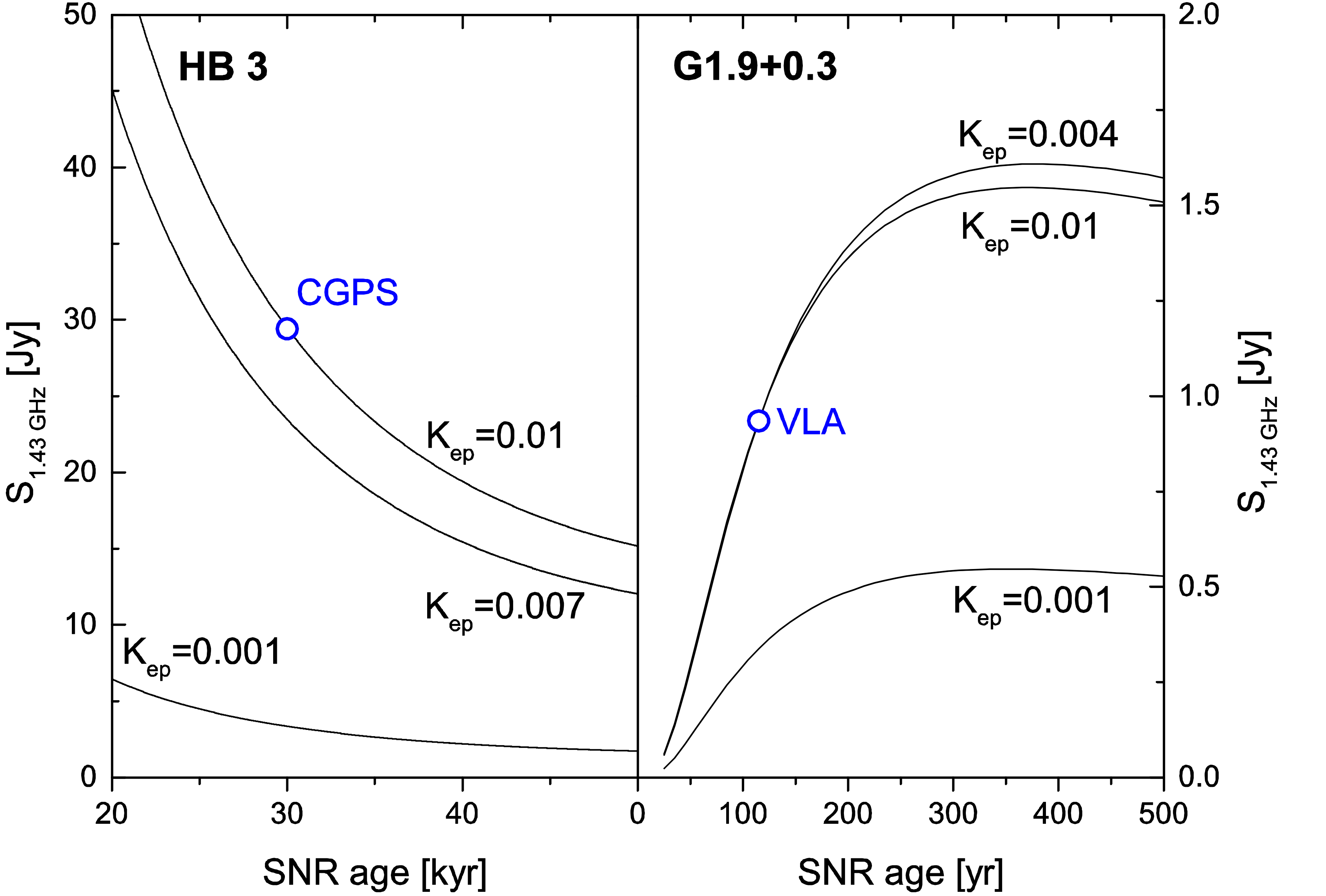
Electron eqp

$$\varepsilon_{\mathrm{CR}} = K_e (m_e c^2)^{2-\gamma} \frac{\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})}{2\sqrt{\pi}(\gamma-1)}(1+\kappa)$$

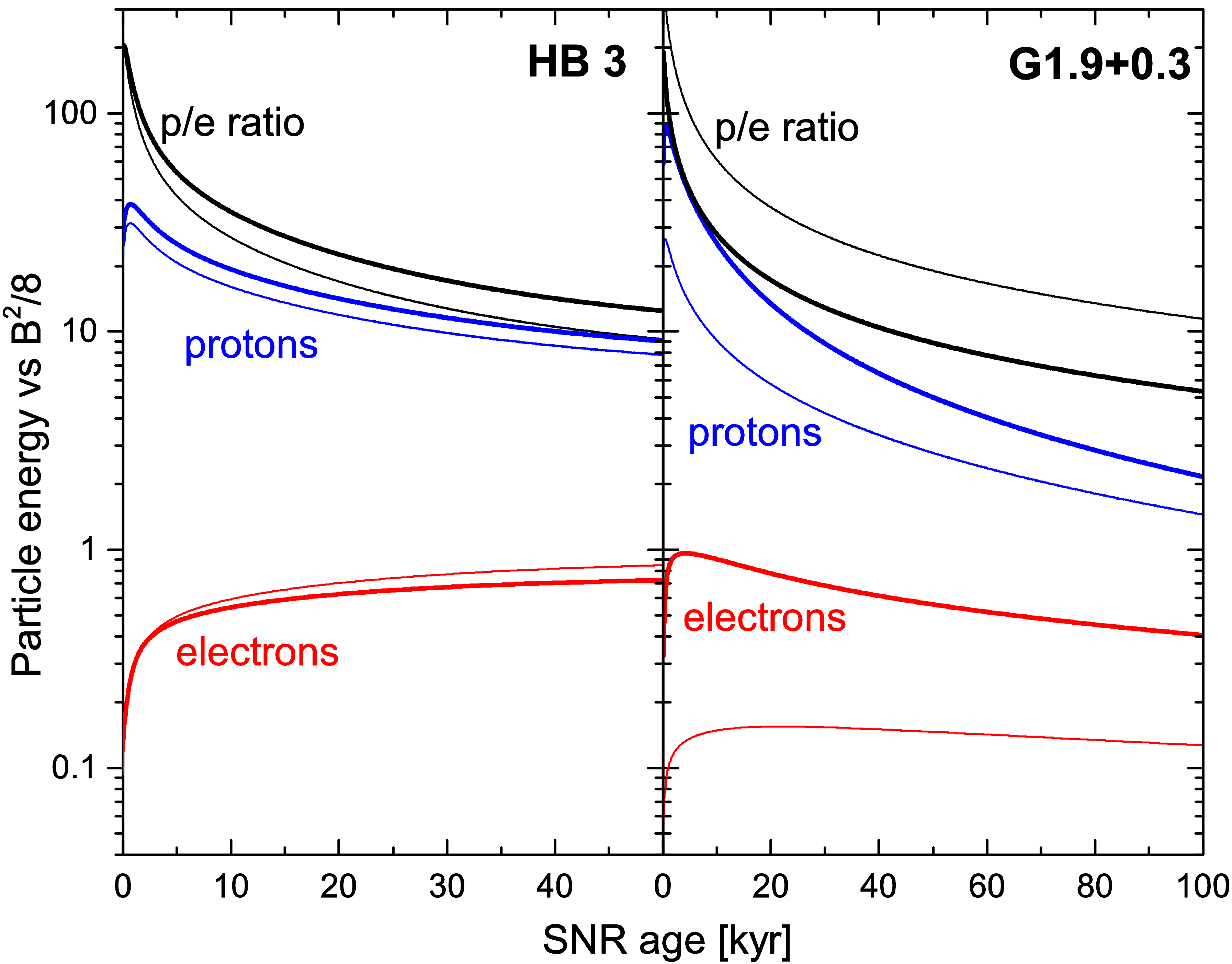
$$\varepsilon_{\mathrm{e}} = K_{\mathrm{e}} (m_{\mathrm{e}} c^2)^{2-\gamma} \frac{\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})}{2\sqrt{\pi}(\gamma-1)}$$

Electron eqp

$$
\begin{aligned}
B\ [\mathrm{Ga}] \ &\approx\ \Bigg(6.286\cdot 10^{(9\gamma-79)/2}\frac{\gamma+1}{\gamma-1}\frac{\Gamma(\frac{3-\gamma}{2})\Gamma(\frac{\gamma-2}{2})\Gamma(\frac{\gamma+7}{4})}{\Gamma(\frac{\gamma+5}{4})}(m_\mathrm{e}c^2)^{2-\gamma}\cdot \\
&\cdot \quad \frac{(2c_1)^{(1-\gamma)/2}}{c_5}\frac{S_\nu[\mathrm{Jy}]}{f\ d[\mathrm{kpc}]\ \theta[\mathrm{arcmin}]^3}\nu[\mathrm{GHz}]^{(\gamma-1)/2}\Bigg)^{2/(\gamma+5)},
\end{aligned}
$$



HB 3
G1.9+0.3
CGPS
VLA
K_{ep}=0.01
K_{ep}=0.007
K_{ep}=0.001
K_{ep}=0.004
K_{ep}=0.01
K_{ep}=0.001
$S_{1.43\ GHz}$ [Jy]
SNR age [kyr]
SNR age [yr]


HB 3
G1.9+0.3
p/e ratio
p/e ratio
protons
protons
electrons
electrons
Particle energy vs B²/8
SNR age [kyr]
100
10
1
0.1
0
10
20
30
40
0
20
40
60
80
100

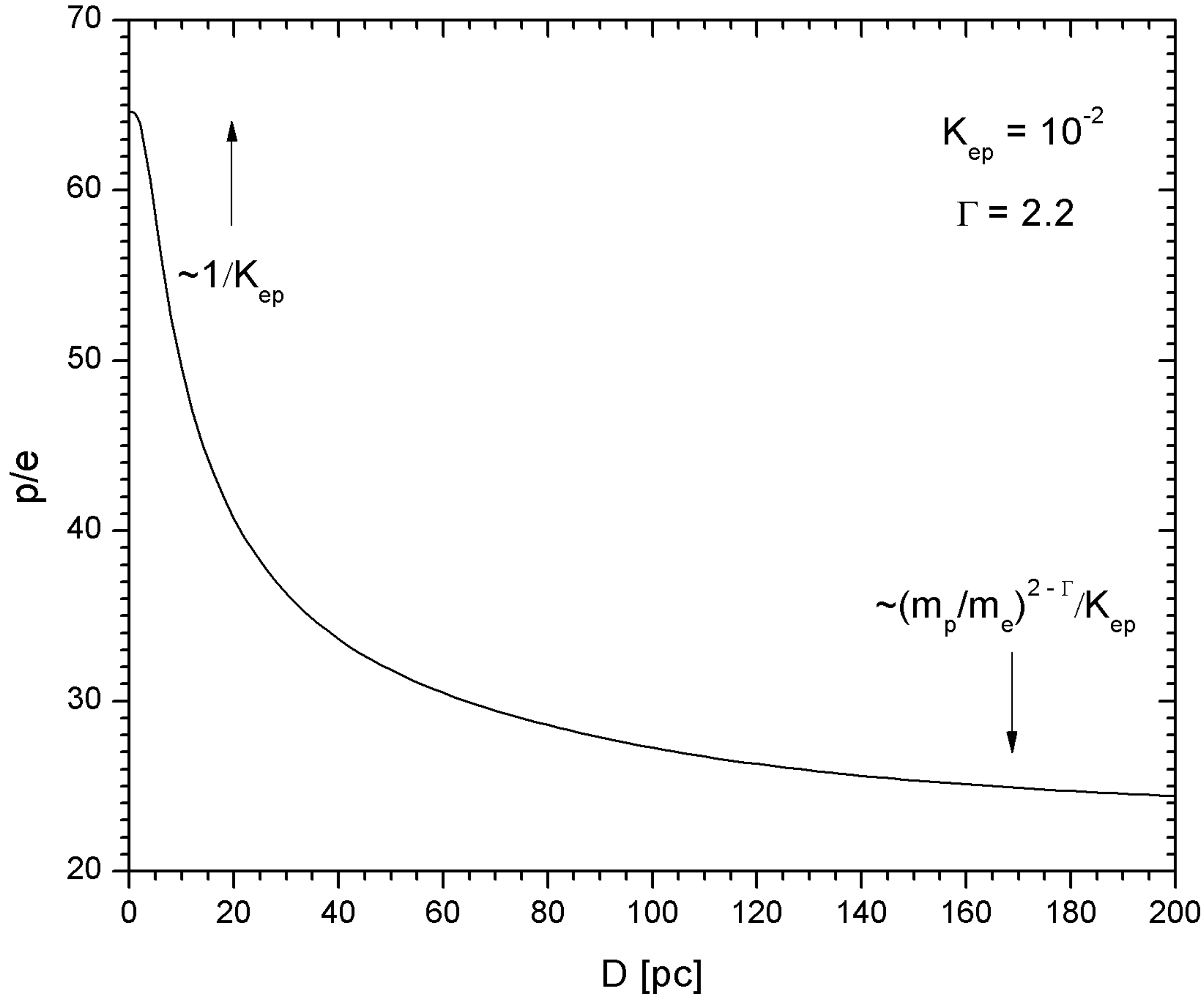
Conclusions

- Eqp is a justified assumption especially between the CR electrons and the magnetic fields in evolved SNRs in the Sedov phase of evolution ($\varepsilon_e/\varepsilon_B$ ~ 0.5)
- We provide evidence suggesting that electron eqp formulae should be used for calculation of the magnetic field strengths in SNRs. The obtained values are approximately 2.5 times lower than those determined in earlier calculations
- Evolved SNRs, especially those embedded in a rarified ambient medium, at the end of the Sedov phase of evolution can reach eqp between CRs and magnetic fields similar to those in the ISM ($\varepsilon_{CR}/\varepsilon_B$ ~ 2)

+ one more result
(may be the most important)

DO WE CATCH ACCELERATION OF ELECTRONS?

Ratio between the energy densities of protons and electrons



Belgrade SNR group

Better part…

THANK YOU VERY MUCH FOR YOUR ATTENTION!!!