The finite groups of random walks in the quarter plane and 4-bar links

Vladimir Dragović

The University of Texas at Dallas/MISANU
e-mail: Vladimir.Dragovic@utdallas.edu

Abstract. We consider maximally space homogeneous random walks as a class of discrete time homogeneous Markov chains, with the state space being the quarter plane. The generators of the process in this region are $\{p_{ij} | -1 \leq i, j \leq 1\}$, where p_{ij} is the transition probability for the jump from (r, s) to $(r + i, s + j)$, for $rs > 0$. This leads to the consideration of a biquadratic curve $C(P)$ in the plane $P^1 \times P^1$, given by its affine equation in $\mathbb{C}^2$:

$$C(P)_A : Q_P(x, y) = xy\Big(\sum_{i,j=-1}^{1} p_{ij}x^i y^j - 1\Big) = 0, p_{ij} \geq 0, \sum_{i,j=-1}^{1} p_{ij} = 1.$$

There is the vertical and horizontal switch, v and h defined on the biquadratic curve $C(P)$. The group of random walk in the quarter plane $H(P)$ is isomorphic to the group of automorphisms of the curve, generated with these two switches:

$$H(P) := < h, p | h^2 = Id, p^2 = Id >,$$

We describe the situations with finite groups of random walks.

Then, we describe periodic 4-bar link configurations.

This is a work in progress, joint with Milena Radnović.

Keywords: random walks in the quarter plane; biquadratic equations; elliptic curves; groups of random walks; periodic 4-bar links.