

Around the Boundary Point Principle

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Abstract. One of the main tool in the qualitative analysis of solutions of elliptic and parabolic equations is the boundary point principle (also known as the Hopf-Oleinik lemma or the normal derivative lemma). It plays a key role in the proof of uniqueness theorems for boundary-value problems and also has some other applications.

This principle states that *a supersolution of a partial differential equation with a minimum value at a boundary point, must increase linearly away from its boundary minimum provided the boundary is smooth enough.*

In the talk we describe the history and discuss the current state of this powerful statement. The talk is based on the results of the papers [1-4].

Keywords: normal derivative lemma; Hopf-Oleinik lemma; boundary point principle.

References

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