

Integrable billiard systems in homogeneous media separated by confocal quadrics

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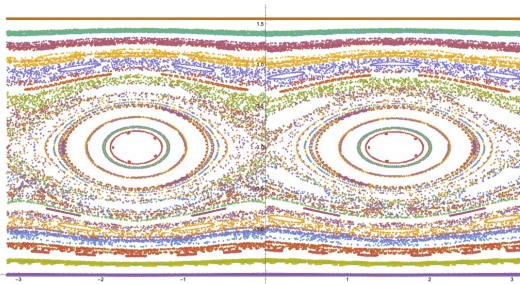
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Abstract. In this talk we focus on a new integrable system related to billiard systems in domains bounded by confocal quadrics.

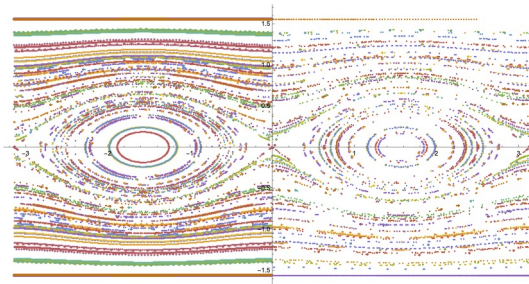
Consider a flat domain bounded by an ellipse that contains two media of different optical densities. The boundary interface between the media is a confocal quadric. At this interface, a light ray is either refracted according to Snell's law or experiences total internal reflection. It is easy to verify experimentally that such a system is not integrable.

However, if we modify the refraction law by replacing the sines of the incident and refracted rays with cosines, then this new system acquires a first integral.

More general systems containing several media separated by confocal quadrics are also integrable. In some of these more general cases, this first integral takes its values on a circle, that is its value is defined up to the addition of a specific, well-defined constant.



Snell law, no integrability



Cosine law, integrable

Explicit formulas for these first integrals in different situations will be presented, along with a description of the surfaces of constant level of the first integral and their bifurcations near the critical value of the first integral.

The talk is based on the paper [1].

Keywords: integrable system, refraction law, confocal quadric

References

- [1] M.A.Nikulin, Th.Yu.Popelensky. Integral level surfaces for the billiard system with cosine refraction. *Sbornik: math*, 2025, 216:10, 101–158.