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Shifted Hardy inequalities on the unit ball and polydisk

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Abstract. We refine and extend shifted Hardy inequalities in several complex variables. A sharp estimate for homogeneous holomorphic polynomials on the sphere in $\mathbb{C}^n$, combined with Yang's one-variable inequality and a slice identity, yields ball inequalities for $H^1(\mathbb{B}^n)$, including a Bergman-scale family with weights. We also prove product inequalities on $\mathbb{D}^n$ and mixed-norm extensions for radial measures, where coefficient weights are moments of the measure. Motivated by Yang's shifted Hilbert form, we define a linear transform, derive its integral representation, compute its exact $H^p(\mathbb{B}^n) \to H^p(\mathbb{B}^n)$ norm, and obtain an $H^1(\mathbb{B}^n) \to H^\infty(\mathbb{B}^n)$ bound.

Keywords: Hardy inequality; Hardy space; homogeneous polynomials; mixed-norm spaces.

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