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Analytic Capacities and Besov Norms of Blaschke Products

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Abstract. Let X be a Banach space of functions analytic in the unit disc $\mathbb{D}$, and let $\sigma = (\lambda_1, \dots, \lambda_N) \in (\mathbb{D} \setminus \{0\})^N$. The quantity

$$\operatorname{cap}_X(\sigma) = \inf\big\{\|f\|_X : f(0) = 1,\ f|_\sigma = 0\big\}$$

is called the X-capacity of the set σ. In this talk, we discuss estimates of the analytic capacities of finite sets in the Besov spaces $\mathcal{B}^0_{p,q}$, equipped with the seminorm

$$\|f\|_{\mathcal{B}^0_{p,q}} = \left(\int_0^1 (1-\rho)^{q-1} \|f'_\rho\|^q_{L^p(\mathbb{T})}\, \mathrm{d}\rho\right)^{1/q},$$

where $f_\rho(z) = f(\rho z)$. These estimates are motivated by the problem of estimating the norm of an inverse matrix in terms of its spectrum for various classes of matrices (including contractions and power-bounded matrices). Our estimates improve inequalities obtained by N. K. Nikolski in 2005 and are optimal for a range of parameters p and q.

The talk is based on joint work with Ilghiz Kayumov (Saint Petersburg), Michael Hartz (Saarbrücken), and Rachid Zarouf (Marseille).