

Almost Calabi-Yau with torsion 6-manifolds, parallel torsion and the instanton condition

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Abstract. It is shown in [2] that on a 6-dimensional compact almost complex Calabi-Yau with torsion (ACYT) 6 dimensional manifold, i.e. an almost complex 6-manifold with an $SU(3)$ -structure which is parallel with respect to the unique connection ∇ with skew-symmetric torsion, the Nijenhuis 3-form is parallel with respect to the torsion connection.

It follows from [3, Lemma 3.4] that if the torsion 3-form of an ACYT $2n$ -manifold is parallel with respect to the torsion connection with holonomy contained in the group $SU(n)$ with Lie algebra $\mathfrak{su}(n)$, then its curvature is an $SU(n)$ instanton, i.e. $R \in \mathfrak{su}(n) \otimes \mathfrak{su}(n)$. This is because $R \in S^2\Lambda^2$ if and only if ∇T is a 4-form, i.e. the 3-form T is Killing.

For the converse, it is observed in [1], that on a compact almost complex Calabi-Yau with torsion (ACYT) 6-manifold with co-closed Lee form the curvature of the torsion connection is an $SU(3)$ -instanton if and only if the torsion is parallel with respect to the torsion connection. The same conclusion holds for any (non necessarily compact) balanced ACYT 6-manifold. In particular, on a CYT 6-manifold (ACYT with vanishing Nijenhuis tensor) the Strominger-Bismut connection is an $SU(3)$ -instanton if and only if the torsion is parallel with respect to the Strominger-Bismut connection provided either the CYT 6-manifold is compact with co-closed Lee form or it is a balanced CYT 6-manifold.

Keywords: torsion connection, $SU(3)$ holonomy, almost Calabi-Yau with torsion, $SU(3)$ -instanton.

References

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