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Effective Computational Models for Flows in Porous Media with Memory

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Abstract. The work considers the problem of unsteady flow of an incompressible fluid in a medium with periodic heterogeneities. At the microscale (pore scale), the flow is described by the unsteady Stokes system with a small parameter corresponding to the ratio of scales, with no-slip conditions on the boundary of the solid skeleton and zero initial conditions. The periodic cell consists of the fluid region, the solid phase, and the fluid–solid interface; periodicity conditions are imposed on the outer boundary of the cell.

The key scientific challenge is accounting for the microstructure of the medium, high computational costs, strong parameter heterogeneity, and the need for multiscale methods. In the case of high-contrast media, standard homogenization fails: memory effects arise, and the effective macroscopic model becomes integro-differential in nature. At the macroscale, the flow is described by Darcy's law with memory, in which the dynamic permeability tensor is determined from the solution of unsteady problems on the periodicity cell (results of G. Allaire, A. Mikelic, G. V. Sandrakov).

It is shown that the components of the dynamic permeability tensor are expressed through the solution of an auxiliary cell problem and represent a sum of exponentials decaying in time. The properties of the tensor are established: symmetry, positive definiteness, and exponential decay. Its connection with the steady-state permeability tensor, obtained as the time integral of the memory kernel, is demonstrated.

To overcome the computational difficulties a transition to local formulations is proposed based on approximating the memory kernel by a finite sum of exponentials. This makes it possible to reduce the integro-differential problem to an extended system with additional functions satisfying first-order ordinary differential equations with corresponding initial conditions. For the resulting macroscopic problem, an a priori stability estimate is formulated under the natural assumption of positive definiteness of the tensor.

The computational algorithm comprises three successive stages [1,2]: solving cell problems on the periodicity cell for the steady-state permeability tensor; solving a partial spectral problem in the fluid domain and calculating the components of the dynamic permeability tensor; and solving the homogenized problem in the computational domain.

Thus, the work formulates the problem of homogenization of unsteady flows in a periodic porous medium, justifies the need to account for memory effects in high-contrast media, constructs an effective macroscopic model with exponential-kernel approximation, investigates its stability, and proposes a complete three-stage computational algorithm with numerical experiments.

Keywords: unsteady flows in porous media; memory Darcy law; integro-differential equation; exponential sum approximation; two-level time discretization schemes.

References

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