Some Applications of Quasidifferentiability in Nonsmooth Optimization

Vivek Laha

Department of Mathematics, Institute of Science, Banaras Hindu University, Varanasi 221005, India
e-mail: laha.vivek333@gmail.com; viveklaha.math@bhu.ac.in
ORCID: 0000-0002-0584-9500

Abstract. This talk discusses optimization problems involving quasidifferentiable functions [1]. A function $h : \mathbb{R}^n \to \mathbb{R}$ is directionally differentiable at $x \in \mathbb{R}^n$ in the direction $d \in \mathbb{R}^n$ if the finite limit

$$h'(x;d) = \lim_{\alpha \downarrow 0} \frac{h(x+\alpha d) - h(x)}{\alpha}$$

exists. It is directionally differentiable at x if this holds for every $d \in \mathbb{R}^n$. The function h is quasidifferentiable at x if it is directionally differentiable there and there are nonempty compact convex sets $\underline{\partial} h(x), \overline{\partial} h(x) \subset \mathbb{R}^n$ such that

$$h'(x;d) = \max_{v \in \underline{\partial} h(x)} \langle v, d\rangle + \min_{w \in \overline{\partial} h(x)} \langle w, d\rangle.$$

The sets $\underline{\partial} h(x)$ and $\overline{\partial} h(x)$ are called the subdifferential and superdifferential, respectively, and $Dh(x) = [\underline{\partial} h(x), \overline{\partial} h(x)]$ is a quasidifferential of h at x.

We present optimality conditions arising from recent work on fractional programming [2] and variational inequalities [3] in terms of quasidifferentials. In particular, the talk addresses Fritz–John and Karush–Kuhn–Tucker type necessary optimality conditions. Further applications concern mathematical programs with equilibrium constraints [4] and vanishing constraints [5].

Keywords: nonsmooth analysis; quasidifferentiability; constraint qualifications; optimality conditions; generalized convexity.

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