

Is there a proper way to define a sectional curvature on a generalized Riemannian space in Eisenhart's sense?

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Abstract. The talk is partially based on the talk that the author gave at RIGA 2025 conference [1]. The motivation to study generalized Riemannian spaces in Eisenhart's sense came from the monograph [5]. On the other hand, A. Mihai and I. Mihai [2], inspired by B. Opozda's papers [3] and [4], defined the $(0,4)$ tensor field S , by $S(X, Y, Z, W) := \frac{1}{2}[g(R(X, Y)W, Z) + g(R'(X, Y)W, Z)] = \frac{1}{2}[g(R(X, Y)W, Z) - g(R(X, Y)Z, W)]$. The tensor S served for the proper definition of sectional curvature of a semi-symmetric non-metric connection, by $S(e_1, e_2, e_1, e_2)$, which does not depend on the orthonormal basis e_1 and e_2 of the tangent space $T_p M$ at the point p of manifold M . Our aim is to well-define the sectional curvature of a linear connection determined by generalized Koszul's formula on generalized Riemannian manifold in Eisenhart's sense. In that case, there are different tensor fields that "play the role," in some sense, of the curvature, but lack some of usual curvature properties. We ask the question from the title of the talk and try to find the right path to study such a problem.

Keywords: linear connection; torsion; manifold; curvature; sectional curvature.

References

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