

Is there a proper way to define a sectional curvature on a generalized Riemannian space in Eisenhart's sense?

Miloš Z. Petrović

University of Niš, Faculty of Agriculture in Kruševac, Kosačićeva 4, 37000 Kruševac, Republic of Serbia
e-mail: petrovic.milos@ni.ac.rs
ORCID: 0000-0002-0308-8074

Abstract. The talk is partially based on the talk that the author gave at RIGA 2025 conference [1]. The motivation to study generalized Riemannian spaces in Eisenhart's sense came from the monograph [5]. On the other hand, A. Mihai and I. Mihai [2], inspired by B. Opozda's papers [3] and [4], defined the $(0, 4)$ tensor field S , by $S(X, Y, Z, W) := \frac{1}{2}[g(R(X, Y)W, Z) + g(R'(X, Y)W, Z)] = \frac{1}{2}[g(R(X, Y)W, Z) - g(R(X, Y)Z, W)]$. The tensor S served for the proper definition of sectional curvature of a semi-symmetric non-metric connection, by $S(e_1, e_2, e_1, e_2)$, which does not depend on the orthonormal basis e_1 and e_2 of the tangent space $T_p M$ at the point p of manifold M . Our aim is to well-define the sectional curvature of a linear connection determined by generalized Koszul's formula on generalized Riemannian manifold in Eisenhart's sense. In that case, there are different tensor fields that "play the role," in some sense, of the curvature, but lack some of usual curvature properties. We ask the question from the title of the talk and try to find the right path to study such a problem.

Keywords: linear connection; torsion; manifold; curvature; sectional curvature.

References

- [1] **M. Z. Petrović.** On sectional curvature of generalized Riemannian spaces in Eisenhart's sense. In: *Book of abstracts, International Conference Riemannian Geometry and Applications – RIGA 2025*. Technical University of Civil Engineering Bucharest, Bucharest, 2025, 30–30. https://fcfdp.utcb.ro/wp-content/uploads/2025/05/Book-of-Abstracts_final.pdf
- [2] **A. Mihai, I. Mihai.** A note on a well-defined sectional curvature of a semi-symmetric non-metric connection. *International Electronic Journal of Geometry*, 2024, 17, 15–23.
- [3] **B. Opozda.** Bochner's technique for statistical structures. *Annals of Global Analysis and Geometry*, 2015, 48, 357–395.
- [4] **B. Opozda.** A sectional curvature for statistical structures. *Linear Algebra and Its Applications*, 2016, 497, 134–161.
- [5] **S. M. Minčić, M. S. Stanković and Lj. S. Velimirović.** Generalized Riemannian spaces and spaces of non-symmetric affine connection. *Monograph, 1st ed.*, University of Niš, Faculty of Sciences and Mathematics, Niš, 2013.